

# COMPILERS

## Liveness Analysis

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hussein suleman  
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### Register Allocation

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- ❑ IR trees are tiled to determine instructions, but registers are not assigned.
- ❑ Can we assign registers arbitrarily?
- ❑ What if:

```
mov d0, 24
mov d1, 36
add d0, d1
```
- ❑ was translated to:

```
mov eax, 24
mov eax, 36
add eax, eax
```

## Allocation Issues

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- Issues:
  - Registers already have previous values when they are used.
  - But there are a limited number of registers so we have to reuse!
  - Use of particular registers affects which instructions to choose.
  - Register vs. memory use affects the number and nature of LOAD/STORE instructions.
- Optimal allocation of registers is difficult.
  - NP-complete for  $k > 1$  registers

## Liveness Analysis

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- Problem:
  - IR contains an unbounded number of temporaries.
  - Actual machine has bounded number of registers.
- Approach:
  - Temporaries with disjoint live ranges (where their values are needed) can map to same register.
  - If not enough registers then spill some temporaries.
    - [i.e., keep them in memory]
- Liveness Analysis = determining when variables/registers hold values that may still be needed.

## Control Flow Analysis

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- Before performing liveness analysis, we need to understand the control flow by building a control flow graph [CFG]:
  - Nodes may be individual program statements or basic blocks.
  - Edges represent potential flow of control.
- Out-edges from node  $n$  lead to successor nodes,  $succ[n]$ .
- In-edges to node  $n$  come from predecessor nodes,  $pred[n]$ .

## Control Flow Example 1/2

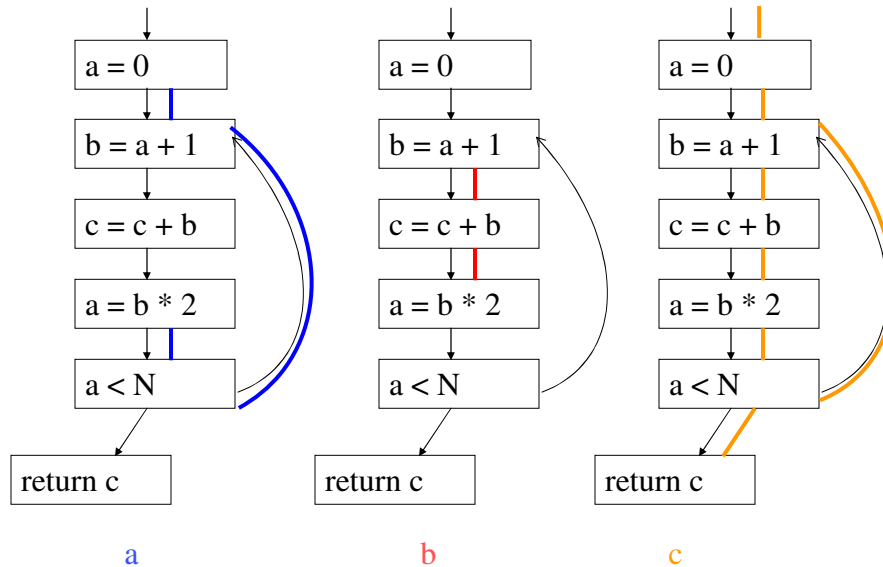
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### □ Sample Program

```

a = 0
L1:  b = a + 1
     c = c + b
     a = b * 2
     if a < N goto L1
     return c
```

## Control Flow Example 2/2



## def and use

- Gathering liveness information is a form of data flow analysis operating over the CFG.
- Liveness of variables “flows” around the edges of the graph.
  - assignments define a variable,  $v$ :
    - $\text{def}[v]$  set of graph nodes that define  $v$
    - $\text{def}[n]$  set of variables defined by node  $n$
  - occurrences of  $v$  in expressions use it:
    - $\text{use}[v]$  = set of nodes that use  $v$
    - $\text{use}[n]$  = set of variables used in node  $n$

## Calculating Liveness 1/3

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- $v$  is *live* on edge  $e$  if there is a directed path from  $e$  to a use of  $v$  that does not pass through any  $\text{def}[v]$ .
- $v$  is *live-in* at node  $n$  if live on any of  $n$ 's in-edges.
- $v$  is *live-out* at  $n$  if live on any of  $n$ 's out-edges.
- $v \in \text{use}[n] \Rightarrow v$  live-in at  $n$
- $v$  live-in at  $n \Rightarrow v$  live-out at all  $m \in \text{pred}[n]$
- $v$  live-out at  $n, v \notin \text{def}[n] \Rightarrow v$  live-in at  $n$

## Calculating Liveness 2/3

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- Define:
  - $\text{in}[n]$ : variables live-in at  $n$
  - $\text{out}[n]$ : variables live-out at  $n$
- Then:
  - $\text{out}[n] = \bigcup_{s \in \text{succ}[n]} \text{in}[s]$
  - for a single node successor,
    - $\text{succ}[n] = \{\}$   $\Rightarrow$   $\text{out}[n] = \{\}$

## Calculating Liveness 3/3

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- Note:
  - $in[n] \supseteq use[n]$
  - $in[n] \supseteq out[n] - def[n]$
- $use[n]$  and  $def[n]$  are constant  
[independent of control flow]
- Now,
- $v \in in[n]$  iff  $v \in use[n]$  or  $v \in out[n] - def[n]$
- Thus,  $in[n] = use[n] \cup [out[n] - def[n]]$

## Iterative Liveness Calculation Algorithm

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```
foreach n {in[n] = {}; out[n] = {}}
repeat
  foreach n
    in'[n] = in[n];
    out'[n] = out[n];
    in[n] = use[n]  $\cup$  (out[n] - def[n])
    out[n] =  $\cup_{s \in succ[n]} in[s]$ 
until  $\forall n (in'[n] = in[n]) \ \&$ 
      (out'[n] = out[n])
```

## Liveness Algorithm Notes

- Should order computation of inner loop to follow the “flow”.
  - Liveness flows backward along control-flow arcs, from out to in.
- Nodes can just as easily be basic blocks to reduce CFG size.
- Could do one variable at a time, from uses back to defs, noting liveness along the way.

## Liveness Algorithm Complexity 1/2

- Complexity: for input program of size  $N$ 
  - $\leq N$  nodes in CFG
  - $\Rightarrow < N$  variables
  - $\Rightarrow N$  elements per in/out
  - $\Rightarrow O(N)$  time per set-union
- for loop performs constant number of set operations per node
  - $\Rightarrow O(N^2)$  time for for loop

## Liveness Algorithm Complexity 2/2

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- Each iteration of repeat loop can only add to each set.
- Sets can contain at most every variable
- $\Rightarrow$  sizes of all in and out sets sum to  $2N^2$ ,
- bounding the number of iterations of the repeat loop
- $\Rightarrow$  worst-case complexity of  $O(N^4)$
- ordering can cut repeat loop down to 2-3 iterations
- $\Rightarrow O(N)$  or  $O(N^2)$  in practice

## Optimality 1/2

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- Least fixed points
  - There is often more than one solution for a given dataflow problem (see example in text).
  - Any solution to dataflow equations is a conservative approximation:
    - $v$  has some later use downstream from  $n$ ,
      - $\Rightarrow v \in \text{out}(n)$
      - but not the converse
- What is the implication of a non-least-fixed-point?



## Optimality 2/2

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- ❑ Conservatively assuming a variable is live does not break the program; just means more registers may be needed.
- ❑ Assuming a variable is dead when it is really live will break things.
- ❑ May be many possible solutions but want the “smallest”: the least fixed point.
- ❑ The iterative liveness computation computes this least fixed point.